Phenomenology and Philosophy of Arithmetic

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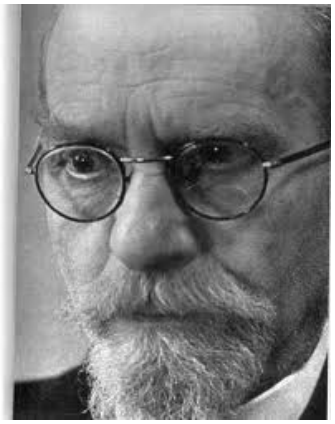

References

1. Edmund Husserl, *Philosophy of Arithmetic: Psychological and Logical Investigations - with Supplementary Texts from 1887-1901* (Dallas Willard's translation).
2. Gottlob Frege, *Review of Dr. E. Husserl's Philosophy of Arithmetic*, Mind New Series, Vol. 81, No. 323 (1972), pp. 321-337 Oxford University Press.
3. Stefania Centrone, *Logic and Philosophy of Mathematics in the Early Husserl* . Synthese Library 345. Dordrecht: Springer, 2010.

Forthcoming issue of The New Yearbook for Phenomenology and Phenomenological Philosophy. Reviews:

1. Stefania Centrone, Logic and Philosophy of Mathematics in the Early Husserl.
2. Mirja Hartimo: Making Sense of Husserls Early Writings on Mathematics.
3. Robert Tragesser : Commentary on Some Themes in Centrones Logic and Philosophy in the Early Husserl.
4. Stefania Centrone: Mathematical Existence, Mathematical Fictions, Etiological Proofs and Other Matters: Reply to Hartimo and Tragesser.
5. Mark van Atten: Stefania Centrone, Logic and Philosophy of Mathematics in the Early Husserl.
6. Stefania Centrone: Reply to Mark van Atten.
7. Burt C. Hopkins,The Philosophy of Husserl.
8. Steven Crowell: Retrieveing Husserls Phenomenology: Hopkins on Philosophys Last Stand.

…AND ONE MORE REFERENCE

Frank Quinn, *A Revolution in Mathematics? What Really Happened a Century Ago and Why It Matters Today*, Notices of the AMS, December 2012.

> *The main point of this article is not that a revolution occurred, but that there are penalties for not being aware of it....*
> *Strangely, mathematicians are also unaware that their field changed so profoundly. Newcomers found philosophical arguments incomprehensible and irrelevant, and philosophy went from a respectable pursuit to an object of ridicule and evidence of senility in just a few decades. But this replaced bad understanding with no understanding at all.*

Some motivation: Gödel on phenomenology

From *The modern development of the foundations of mathematics in the light of philosophy* (1961):

A general schema of possible philosophical world-views:

(Left)	(Right)
Skepticism	Spiritualism
Materialism	Theology
Positivism	Idealism
...	...

Hilbert's program in the middle

"As far and the rightness or wrongness, or, respectively, truth and falsity, of these two directions is concerned, the correct attitude appears to me to be that the truth lies in the middle or consists of the combination of the two conceptions.

Now, in the case of mathematics, Hilbert had of course attempted just such a combination, but one obviously too primitive and tending too strongly in one direction. In any case there is no reason to trust blindly in the spirit of the time, and it is therefore undoubtedly worth the effort [[at least]] to try the other of the alternatives mentioned above ... in the hope of obtaining in this way a workable combination."

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Mechanical manipulation vs. knowledge of abstract objects

"Obviously, this means that the certainty of mathematics is to be secured not by proving certain properties by a projection onto material systems—namely, the manipulation of physical symbols—but rather by cultivating (deepening) knowledge of the of the abstract concepts themselves which lead to the settings of these mechanical systems, and further by seeking, according to the same procedures, to gain insights into the solvability, and the actual method of solution, of *all* meaningful mathematical problems."

Gödel on phenomenology

"Now in fact, there exists today the beginning of a science which claims to possess a systematic method for such a clarification of meaning, and that is phenomenology founded by Husserl. Here clarification of meaning consists in focusing more sharply on the concepts concerned by directing our attention in a certain way, namely, onto our own acts in the use of these concepts, onto our own powers in carrying those acts, etc. But one must keep clearly in mind that this phenomenology is not a science in the same sense as the other sciences. Rather it is ... a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other basic concepts hitherto unknown to us."

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More in...

Mark Van Atten, Juliette Kennedy. *On the Philosophical Development of Kurt Gödel*. The Bulletin of Symbolic Logic 9 (4), (2003)

Edmund Husserl (1859-1938)

- 1876-1878, studies mathematics, physics, and astronomy at University of Leipzig.
- 1878-1881, studies mathematics under Leopold Kronecker and Karl Weierstrass at Humboldt University of Berlin.
- 1881-1883, studies mathematics under the supervision of Leo Königsberger (a former student of Weierstrass) at University of Vienna. In 1883 obtains Ph.D. with the work *Contributions to the Calculus of Variations*.
- 1883-1886, returns to Berlin to work as the assistant to Karl Weierstrass.

Edmund Husserl (1859-1938)

- 1884, at the University of Vienna attends the lectures of Franz Brentano on philosophy and philosophical psychology. Brentano introduces him to the writings of Bolzano, Lotze, Mill, and Hume.
- 1886-1987, University of Halle, habilitation thesis *On the concept of Number* serves as the base for *Philosophy of Arithmetic* published in 1891.
- 1887-1901, teaches philosophy at the Martin Luther University of Halle-Wittenberg.
- 1901-1916, professor at the University of Göttingen. *Logical Investigations* published in 1900-1901.

From Husserl's description of PA

"For a deeper philosophical understanding of arithmetic two things are currently necessary: on the one hand, an analysis of its basic concepts; on the other hand a logical illumination of its symbolic methods. ... The First Part includes, in the main, **psychological** investigations concerning the concepts *multiplicity, unity,* and *cardinal number* to the extent that these are not given to us in symbolic (indirect) forms. The second part considers the symbolic representations of multiplicity and and number, and seeks to detect, in the fact that we are almost always confined to symbolic representation of number, the logical origin of a general arithmetic."

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On Philosophy of Arithmetic

In his own words, from *Early Writings* p. 490.
"... How immature, how naive and almost childlike that work appeared to me! Well, it was not without reason that I was conscience-stricken upon its publication. Actually, I had already gone beyond it as I published it. Indeed, it was drawn in essentials from the years 1886 and 1887. I was a novice, without a correct understanding of philosophical problems, without proper exercise of philosophical abilities. And while laboring over projects concerning the logic of mathematical thought, and of the mathematical calculus in particular, I was tormented by those incredibly strange realms: the world of the purely logical and the world of actual consciousness or, as I would say now, that of the phenomenological and also the psychological. I had no idea of how to unite them; and yet they had to interrelate and form an intrinsic unity. ..."

On abstraction

"**No concept can be thought without foundation in a concrete intuition.** Hence, even when we represent the general concept of the multiplicity we always have in consciousness the intuition of some concrete multiplicity by means of which we abstract the general concept. In what way, then, does this abstraction proceed? ... total abstraction from the peculiarities of the individual contents colligated must be effected, retaining however, their combination. This appears to present a difficulty, if not a psychological impossibility. If that abstraction is performed in all seriousness, then of course the collective combination disappears along with the individual contents, instead of remaining behind as a conceptual distillate."

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THE SOLUTION IS CLEAR...

"....To disregard or abstract from something means merely to give it no special notice. The satisfaction of the requirement wholly to abstract from the peculiarities of the contents thus absolutely does not have the effect of making those contents ... disappear from our consciousness. ...
Multiplicity in general—as we can now express ourselves quite simply and without any circumlocution—is nothing other than: a certain something and a certain something and a certain something, etc.; or, some one and some one and some one thing, etc.; or more briefly, one and one and one, etc."

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Cardinal number [Anzahl]

" *Cardinal number* is a common name name for the concepts two, three, four, etc. Now we certainly do also speak of a general concept and not merely of a general name, "cardinal number." Bur we cannot explain this concept otherwise than by pointing to the similarity which all the number concepts have to each other. **There is no cardinal number in general, understood as separately noticable partial representation ... which might be isolated within the representation of each of the cardinal number concepts.**"

On defining

"Ever since Euclid's *Elements* attained the status of model of scientific exposition, mathematicians have followed the principle of not regarding mathematical concepts as fully legitimized until they are well-distinguished by means of rigorous definitions. But the principle, undoubtedly quite useful, has not infrequently and without justification been carried too far. In over zealousness for a presumed rigor, attempts were also made to define concepts that, because of their elemental character, are neither capable of definitions **nor in need of it**. ... These definitions, baseless and scientifically useless, have nevertheless, in virtue of a certain formal character, found favor among mathematicians and among philosophers influenced by them."

Numbers as equivalence classes

"... it results from our analysis, **with incontestable clarity**, that the concepts of multiplicity and of unity rest directly upon ultimate, elemental psychical data, and consequently belong among the concepts that are indefinable in the sense indicated. But the concept of number is so closely joined to them that also in its case one can scarcely speak of any "defining." The goal that Frege sets for himself must therefore be termed chimerical. It is therefore no wonder if his work, in spite of all ingenuity, gets lost in unfruitful super-subtleties and concludes without positive results."

Numbers as equivalence classes

"I am unable to find that this method represents an enrichment of logic. Its results are of the type that can only make us wonder how anyone could even provisionally take them to be correct. In fact, what this method allows us to define are not the contents of the concepts *direction*, *shape* and *number*, but rather their *extensions*."

Authentic and symbolic representations

"If we had authentic representations of all numbers, as we do for the first ones in the series, then there would be no arithmetic, for it would be then completely superfluous. The most complicated relations between numbers ... would then along with the number representations be simultaneously present to us with the same intuitive Evidence as we have when we have with propositions such as 2+3=5. ... But in fact we are extremely limited in our representational capacities. That some sot of limits are imposed upon us here lies in the finitude of human nature. **Only from an infinite understanding can we expect the authentic representation of *all* numbers**; for, surely, therein would ultimately lie the capability of uniting true infinitude of elements into an explicit representation. ...

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Authentic and symbolic representations

... But *how* can one speak of concepts which one does not genuinely have? And *how* is it not absurd that upon such concepts the most secure of all sciences, arithmetic, should be grounded? The answer is: Even if we do not have the concept given in the *authentic* manner, we still do have it given—in the symbolic manner."

Some questions

1. Is there still room for a pre-Hilbertian analysis of basic concepts of mathematics (other than set and membership)?
2. Are we forever committed to the set-theoretic foundations of arithmetic?
3. Does the authentic/symbolic number representation dichotomy matter for foundations of arithmetic?
4. What is arithmetic after all?