Degrees of interpretability of finitely axiomatized sequential theories

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Finitely axiomatized sequential theories are something like a natural kind of theories. They share a lot of salient and important properties. Moreover, many familiar theories belong to this kind. Examples of finitely axiomatized sequential theories are the basic theory PA^-, Buss's theory S^1_2, Elementary Arithmetic EA, $\mathrm{I}\Sigma_1$, ACA_0 and the Gödel-Bernays theory of sets and classes GB.

The study of interpretability degrees of a class of theories is important as the study of a notion of the strength of a theory. For example the Observed Linearity of Reverse Mathematics only comes into focus against the background of the result that the surrounding degree structure contains infinite anti-chains.

In this talk we give an introduction to the interpretability degrees of finitely axiomatized sequential theories. We are especially interested in the question: how are the degrees of extensions (in the same language) of a given theory embedded in the complete degree structure? We will briefly look at the case of a non-sequential theory, to wit: Robinson's Arithmetic Q. This case shows interesting similarities and differences with the sequential one.

As we will see arithmetical theories play a central role in the study of the interpretability degrees of finitely axiomatized sequential theories. This is already visible in the classical result that each such degree contains an arithmetical theory.