Team semantics for logics with generalized quantifiers

Logic seminar 2017, Gothenburg

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April 21, 2017

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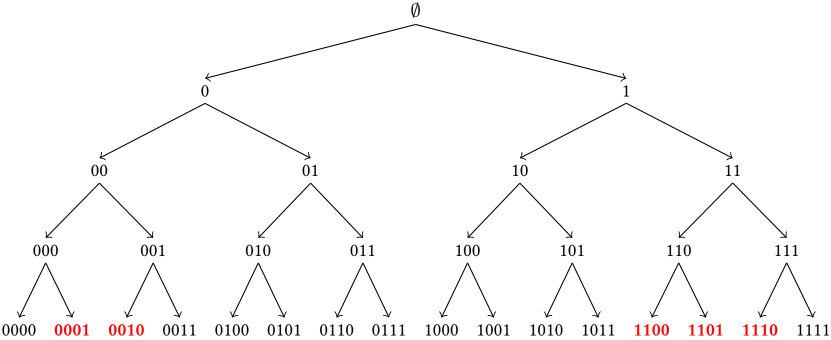

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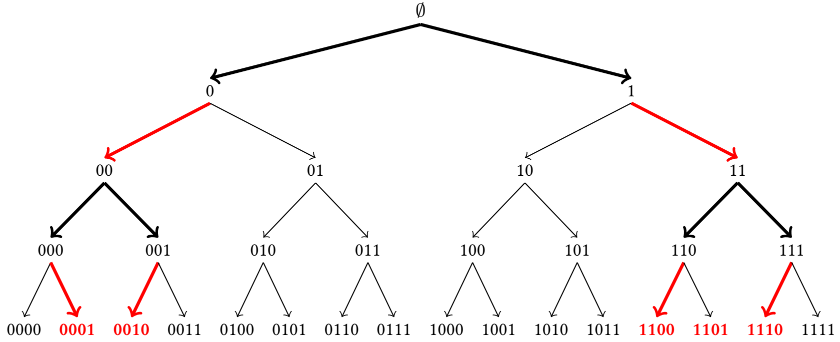

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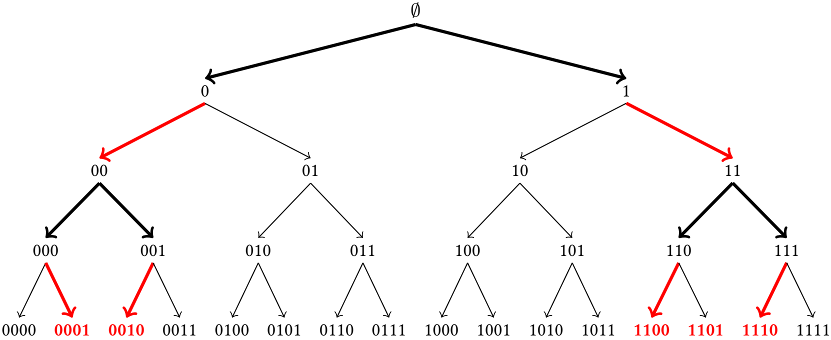

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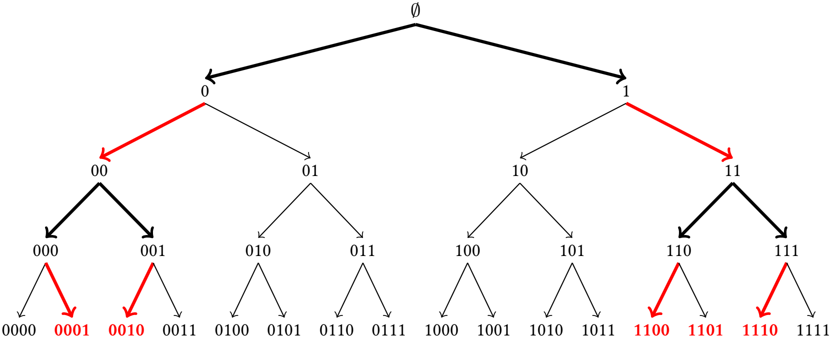

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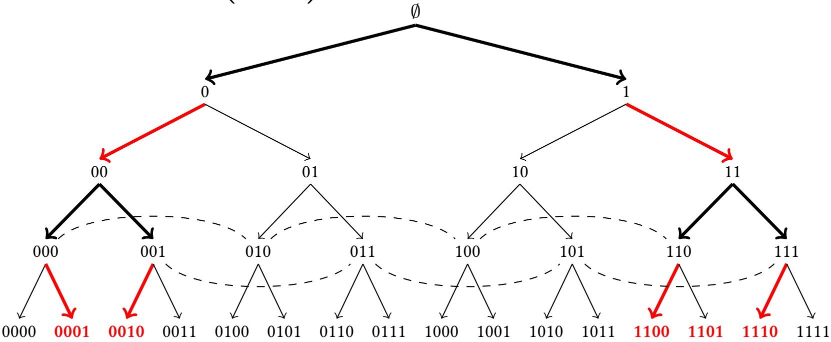

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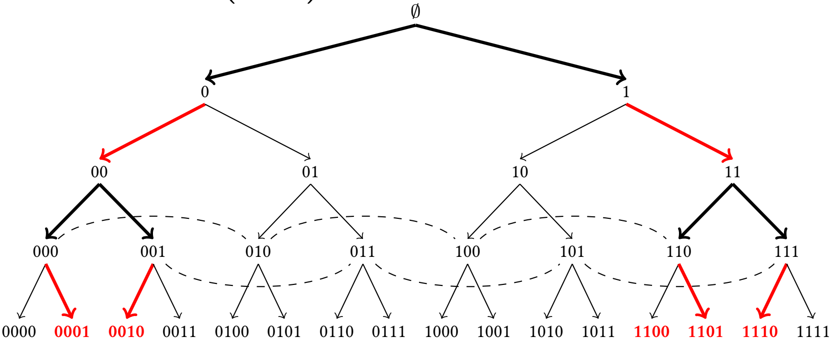

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Definition

X a **team** $=$ set of assignments.

$M, X \models D(\bar{x}, y)$ iff for all $s, s' \in X$ if $s(\bar{x}) = s'(\bar{x})$ then $s(y) = s'(y)$.

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This talk introduces a logic in which flatness fails.

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- **Extra feature of D:** Truth is definable.

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Q is **monotone increasing** if $A \subseteq B$ and $A \in Q_M$ implies $B \in Q_M$.

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Works well only for **monotone increasing generalized quantifiers**.

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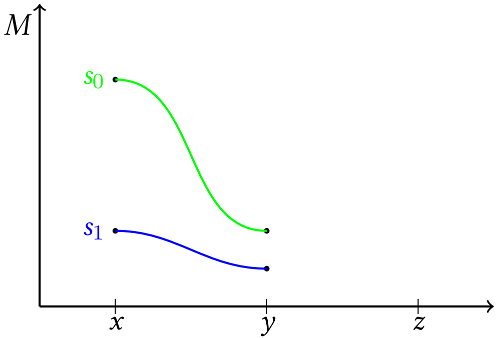

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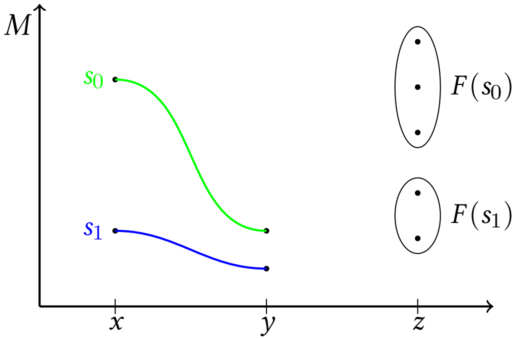

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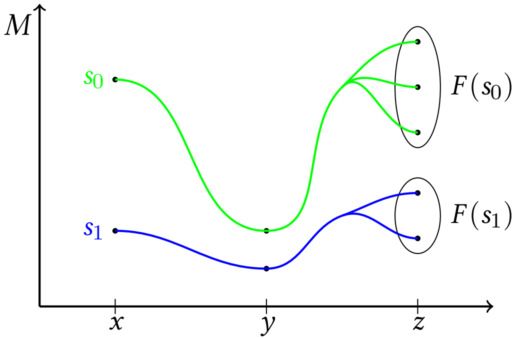

Iteration and branching

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For monotone increasing quantifiers:

$\mathrm{Br}(Q_1, Q_2)_M = \left\{ R \subseteq M^2 \;\middle|\; A \times B \subseteq R, A \in (Q_1)_M, B \in (Q_2)_M \right\}$

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Express branching

$$D(Q) \equiv D(Q, \mathrm{Br}(Q, Q))$$

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Theorem (E., Kontinen, Väänänen)

There are sound and complete inference systems wrt the following consequence relations:

- $\Gamma \vDash_w \phi$ where ϕ is $\mathrm{FO}(Q, \check{Q})$.
- $\Gamma \vDash \phi$ where ϕ is $\mathrm{FO}(Q_1, \check{Q}_1)$.

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- ~~every assignment $s \in X$ satisfies ϕ.~~
- for every assignment $s : \mathrm{dom}(X) \to M^k$, $s \in X$ **iff** s satisfies ϕ.

Dependence logic, take II

$$\phi ::= \mathrm{At} \mid \neg\mathrm{At} \mid D(\bar{x}) \mid \phi \wedge \phi \mid \phi \vee \phi \mid \exists x \phi \mid \forall x \phi$$

- $M, X \vDash \gamma$ if for all $s \in X$: $M, s \vDash \gamma$, where γ is a literal.

$$Qx\,X = \{\, s : \mathrm{dom}(X) \setminus \{\, x \,\} \to M \mid \{\, a \in M \mid s[a/x] \in X \,\} \in Q_M \,\}$$

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Basic principle

A formula ϕ is satisfied by a team X if for every assignment $s : \mathrm{dom}(X) \to M^k$, $s \in X$ iff s satisfies ϕ.

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If $x \in \mathrm{dom}(X) \setminus (\mathrm{fv}(\phi) \cup \mathrm{bv}(\phi))$ and $\mathrm{dom}(X) \cap \mathrm{bv}(\phi) = \emptyset$ then

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Relationship with Dependence Logic

$X \vDash D(\bar{x}, y)$ iff

$$X \vDash \exists z\Big(\forall \bar{w}(\top(\bar{x}, y) \otimes \top(\bar{x}, z)) \land (y = z \otimes \top(\bar{x}, \bar{w}))\Big),$$

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Define $f(\bar{w}, \phi)$ on the set of dependence logic formulas $D[\tau]$:

- $f(\bar{w}, D(\bar{x}, y)) = \exists z(\forall \bar{w}'(\top(\bar{x}, y) \otimes \top(\bar{x}, z)) \wedge (y = z \otimes \top(\bar{x}, \bar{w}')),$ where $\bar{w}' = \bar{w} \setminus \{\bar{x}, y, z\}$ and z is not in $\bar{w}$.
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For every team X and formula ϕ of $D[\tau]$ such that $\mathrm{dom}(X) = \mathrm{fv}(\phi)$:

$$M, X \models_{\mathrm{DL}} \phi \text{ iff } M, X \models \phi^+.$$

Expressive power

We may, in a similar fashion, give a translation g of independence logic into team logic in such a way that

$$M, X \models \phi \text{ iff } M, X \models g(\phi, \mathrm{dom}(X)).$$

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For every $T[\tau]$-formula ϕ there is a Σ^1_1 formula Θ in the language of $\tau \cup \{ R \}$ such that for all M and X: $M, X \vDash \phi$ iff $(M, \mathrm{rel}(X)) \vDash \Theta$.

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The expressive power of team logic is that of existential second-order logic, for both formulas and sentences.

Generalized quantifiers revisited

Let Q be of type $\langle n \rangle$ then $M, X \models Q\bar{x}\phi$ iff there is Y such that $\bar{x} \in \mathrm{dom}(Y)$, $M, Y \models \phi$ and $\exists \bar{x} X = Q\bar{x}Y$, where

$$Q\bar{x}Y = \{ s : \mathrm{dom}(Y) \setminus \{ \bar{x} \} \to M \mid Y_s(\bar{x}) \in Q_M \}.$$

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Conservative over $\mathrm{FO}(Q)$:

For every untangled ϕ formula of $\mathrm{FO}(Q)$ and every team X such that $\mathrm{dom}(X) \cap \mathrm{bv}(\phi) = \emptyset$:

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Respects iteration:

$$M, X \vDash (Q_1 \cdot Q_2)xy\phi \;\text{ iff }\; M, X \vDash Q_1 x\, Q_2 x\, \phi$$

Conclusion

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That's all folks!

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